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you as teachers whether such an analysis does not present the subject to the pupil in a way likely to interest and benefit him.

In conclusion, I wish to say that I am an enthusiast, but not, I hope, a crank. I try to make my pupils think rather than discover, for thinking is of more importance than the mere discovery of theorems, which should be a means, not an end. I do not claim that this method is a panacea for all ills. The slow pupil is still slow, the careless one is still careless, and sometimes the uninterested are still hard to arouse, but in each of these classes I believe that a little more of the divine fire shows. If there is any way to make geometry a student's holiday, I have yet to find it, but by this method, it is at least a live, businesslike, and commonsense subject, and the pupils know that good judgment and sound logic can overcome all its difficulties.

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# INTERESTING WORK OF YOUNG GEOMETERS.

By J. T. Rorer.

## I.

Although that part of geometry commonly studied in our schools has been so thoroughly worked over year after year for so many centuries, nevertheless new and interesting theorems are occasionally discovered. The following theorems which are not found in the text-books commonly used and which probably have not appeared in the journals, were discovered by a second-year high school student.

1. If the segments of the altitudes of any triangle between the vertices of the angles and the orthocenter be perpendicularly bisected, the lines when produced will form a new triangle congruent to the original triangle; also,

2. The orthocenter of the first triangle is the circumcenter of the second, and conversely; also,

3. If the adjacent vertices of the two triangles are connected, an equilateral hexagon having its opposite sides parallel, will be formed.

The above theorems were suggested to the mind of the student who announced them, by a study of concrete geometry in connection with the demonstrative work of the text. The class had performed such routine work as drawing the altitudes, medians and bisectors of a triangle; inscribing and circumscribing circles, etc. The above theorems were the result of independent constructions made by the student and correctly interpreted by him.

The proofs are quite easy and were soon supplied by other members of the class. Many related exercises and theorems may be devised pertaining to the figure involved.

## II.

The problem of trisecting an angle always proves inviting to beginners. The history of this famous problem when brought to their attention sometimes tends more to heighten than to lessen their enthusiasm.

One such young enthusiast invented the following method of trisecting. The construction approximates so closely to the desired trisection that an error can hardly be detecetd if the drawing is made on a scale commonly used in class-room work.

To trisect $\measuredangle\theta$, or its arc $AB$. On chord $AB$ as diameter construct a circle, center $M$. Let $E$ be the intersection of the bisector of $\theta$ and the circle—the intersection toward the vertex. Let $C$ and $D$ be points of trisection of the outer semicircle, *i. e.*, $AC = AM$; draw $EC$ and $ED$; these will approximately trisect arc $AB$; let these points of approximate trisection be $F$ and $G$.

The trigonometry class, with some suggestion as to the method of attack, evolved this method of finding the error of the above construction.

Let $O$ be the vertex of $\angle\theta$; let $H$ be the mid point of the outer semicircle; draw $OF$, one of the approximate trisectors of $\measuredangle\theta$. In $\triangle OEF$, let $\measuredangle OFE = \kappa$, $OF = r$; then

$$\frac{\sin\kappa}{\sin OEF} = \frac{OE}{r}$$

but, since arc $HC = 30°$ by construction, $\measuredangle HEC = 15°$.

$$\therefore \measuredangle OEF = 165° \text{ and } \sin OEF = \sin 15°.$$

Again

$$OE = OM - EM = OM - AM = r\cos\frac{\theta}{2} - r\sin\frac{\theta}{2}.$$

Substituting these values of sin $OEF$ and $OE$ and solving,

$$\sin\kappa = \sin 15^\circ\left[\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right] \qquad (1)$$

Let $\frac{\theta}{3} = \measuredangle AOF + \epsilon$, where $\epsilon$ is the correction to be applied to the first (or third) approximate third of the given angle, or arc, to obtain the true value, then,

$$\measuredangle AOF = \frac{\theta}{3} - \epsilon$$

and

$$\measuredangle FOH = \frac{\theta}{2} - \measuredangle AOF = \frac{\theta}{2} - \left(\frac{\theta}{3} - \epsilon\right) = \frac{\theta}{6} + \epsilon$$

Again,

$$\measuredangle FOH + \kappa = 15^\circ \quad \text{(Exterior angle of } \triangle\, FOE\text{)}$$

$$\therefore \frac{\theta}{6} + \epsilon + \kappa = 15^\circ$$

$$\therefore \epsilon = 15^\circ - \left(\frac{\theta}{6} + \kappa\right) \qquad (2)$$

From equations (1) and (2), the value of $\epsilon$ may easily be computed for any value of $\theta$. The results of such computation are tabulated for each 10° of arc.

| $\theta$ | 10° | 20° | 30° | 40° | 50° | 60° | 70° | 80° | 90° |
|---|---|---|---|---|---|---|---|---|---|
| | −16′.5 | −27′.2 | −32′.7 | −33′.9 | −31′.5 | −26′.2 | −18′.6 | −9′.7 | 0′ |

| $\theta$ | 100° | 110° | 120° | 130° | 140° | 150° | 160° | 170° |
|---|---|---|---|---|---|---|---|---|
| $\epsilon$ | +9′.7 | +18′.6 | +26′.2 | +31′.5 | +33′.9 | +32′.7 | +27′.2 | +16′.5 |

It will be seen that these corrections are about within the limits of accuracy for ordinary class-room " compass and ruler " work—whether pencil, ink or chalk be used.

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